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Research Article

A High-Precision Hybrid Semi-Analytical Technique for Solving Fractional Reaction-Diffusion Models

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ABSTRACT

In this work, we solve a class of nonlinear time-fractional reaction-diffusion equations using the Tantawy Least Squares Method (TLSM), a high-precision hybrid semi-analytical technique. In particular, we study the fractional Allen-Cahn, Newell-Whitehead, and Fisher's equations, which are essential for simulating fluid convection patterns, population genetics, and phase separation. The TLSM offers a clear benefit over conventional variational and perturbation methods by combining the Tantawy technique's iterative power series structure with a least-squares residual error minimization. For these models, we obtain the general recurrence relations under the Caputo fractional derivative operator. Thorough residual error analysis is used to validate the accuracy of the suggested approach. In terms of convergence speed and absolute error, comparative numerical studies show that TLSM performs noticeably better than the Modified Variational Iteration Algorithm-II.

Keywords: Fractional Reaction-Diffusion Equations; Tantawy Least Squares Method; Allen-Cahn Equation; Newell-Whitehead Equation; Fisher's Equation; Caputo Derivative.

MSC 2020: 35R11; 65M22.

1. Introduction

Fractional calculus has completely transformed the mathematical description of challenging physical systems that show anomalous diffusion, memory effects, and hereditary features [1, 2]. Fractional Partial Differential Equations offer a more realistic representation of non-local dynamics in biological systems, signal processing, and viscoelastic materials than traditional integer-order models [3, 4].

Three important nonlinear reaction-diffusion equations are the subject of this investigation. Initially, phase separation in multi-component alloy systems is described by the Allen-Cahn (AC) equation, which is

important to material studies [5]. Secondly, the dynamical behavior in Rayleigh-Bénard convection close to the bifurcation point is modeled by the Newell-Whitehead (NW) equation [6]. Finally, Fisher's equation explains flame propagation in chemical kinetics and the spatial spread of a viable gene in a biological population [7].

In this work, we consider the following nonlinear time-fractional reaction-diffusion equations given by:

$$ {}_0^C D_t^\alpha w(t,x) = \frac{\partial^2 w(t,x)}{\partial x^2} + C_1 w(t,x) + C_2 w^n(t,x),\ 0 < \alpha \le 1, \tag{1} $$

where C_1 and C2 are real constants. ${}_0^C D_t^\alpha w(t,x)$denotes the Caputo time-fractional derivative of order α, and $w(t,x)$ is the unknown function. Three well-known models are produced by equation (1). The fractional Allen-Cahn equation is formed when $n = 3$, $C_1 = 1$ and $C_2 = -1$, which has numerous uses in plasma physics, biology, and quantum mechanics [18]. Equation (1) is the fractional Newell-Whitehead (NW) equation, which is used in many studies to describe the Rayleigh-Bénard convection of binary fluid mixtures if $n = 2$, $C_1 = 1$ and $C_2 = -1$. This equation was reduced to fractional Fisher's equation for $C_1 = 6$ and $C_2 = -6$.

Because of its nonlinearity, equation (1) is difficult to solve analytically. Numerous numerical techniques have been presented to solve these type of problems, such as the modified Variational iteration algorithm-II by Ahmad et al. [10], the Homotopy analysis method [9], and the Adomian decomposition method (ADM) [8]. Despite their effectiveness, these techniques frequently have poor convergence at longer time steps or necessitate the computation of complex polynomials (Adomian or He's polynomials). We use the TLSM, which was recently developed by Hamali et al. [11] (2026), to get over these restrictions. TLSM combines a least-squares error minimization protocol with a systematic power series approach. Applying the TLSM to the particular challenges identified in [10] is what makes this work innovative; it shows that TLSM produces more stable residual norms and smaller absolute errors than the modified Variational iteration algorithm-II method.

2. Preliminaries

In this section, we present the basic definitions of fractional calculus employed in the proposed method.

Definition 2.1. *The Caputo fractional derivative of order α of a function $u(t,x)$ with respect to t is defined as*

$$ I_t^\alpha w(t,x) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1}\, w(\tau,x) d\tau\,,\ t > 0. \tag{2} $$

Definition 2.2. *The Caputo fractional derivative ${}_0^C D_t^\alpha w(t,x)$ of order α is defined by [13] as*

$$ {}_0^C D_t^\alpha w(t,x) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{w^{(n)}(\tau,x)}{(t-\tau)^{\alpha-n+1}}\, d\tau\,, & n-1 < \alpha < n, \\ \frac{\partial^n w(t,x)}{\partial t^n}, & \alpha = n. \end{cases} \tag{3} $$

Definition 2.3. *A key characteristic of the Tantawy technique is how the Caputo derivative affects power functions [11]:*

$$ {}_0^C D_t^\alpha t^\mu = \frac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)} t^{\mu-\alpha},\ \mu \ge \lceil \alpha \rceil. \tag{4} $$

3. Main Results

To illustrate how the methodology is applied, we look at the nonlinear time-fractional partial differential equation in its general form, which is provided by:

$$ {}_0^C D_t^\alpha w(t,x) = \mathcal{L}(w(t,x)) + \mathcal{N}(w(t,x)) + f(t,x),\ 0 < \alpha \leq 1, \tag{5} $$

subject to the initial condition:

$$ w(0,x) = h(x). \tag{6} $$

Here ${}_0^C D_t^\alpha w(t,x)$ denotes the Caputo time-fractional derivative of α-order and $\mathcal{L}$ is a linear spatial operator, $\mathcal{N}$ is a nonlinear operator and $f(t,x)$ is a source term.

The TLSM uses the framework developed by Hamali et al. [11] to answer the general problem (5) by following these methodical steps:

Step 1. We suppose that the solution $w(t,x)$ with respect to t^α, may be developed as an extended power series:

$$ w(t,x) = \sum_{n=0}^{\infty} p_n(x)\, t^{n\alpha} = p_0(x) + p_1(x)\, t^\alpha + p_2(x)\, t^{2\alpha} + \cdots. \tag{7} $$

The zeroth term is explicitly identified as $p_0(x) = h(x)$ using the initial condition (6).

Step 2. We replace the general equation (5) with the alternative equation (7). The following results are obtained through using the Caputo derivative property to the left-hand side:

$$ {}_0^C D_t^\alpha [\sum_{n=0}^{\infty} p_n(x)\, t^{n\alpha}] = \sum_{n=1}^{\infty} p_n(x)\, \frac{\Gamma(n\alpha+1)}{\Gamma((n-1)\alpha+1)} t^{(n-1)\alpha}. \tag{8} $$

$w(x,t)$ is substituted for growing the right-hand side of the equation:

$$ \mathcal{L}(\sum_{n=0}^{\infty} p_n\, t^{n\alpha}) + \mathcal{N}(\sum_{n=0}^{\infty} p_n\, t^{n\alpha}). \tag{9} $$

Step 3. We obtain a recurrence relation by matching the coefficients of similar powers of t (that is, terms of $t^{(k-1)\alpha}$ on both sides of the problem. In terms of the observed $p_0(x)$, this enables us to successively determine the unknown spatial components $p_1(x), p_2(x), p_3(x), \ldots$

Step 4. We describe the Residual function $Res(t,x)$ as the flaw produced when the approximate solution is re-inserted into the initial determining equation (5) for an approximation solution $w_k(t,x)$ truncated at k terms. This is provided by:

$$ Res(t,x) = {}_0^C D_t^\alpha w_k(t,x) - \mathcal{L}(w_k) - \mathcal{N}(w_k). \tag{10} $$

The exact response should ideally result in $Res(t,x) = 0$. The residual is not zero for the reduced series, though. By reducing the square of the residual over the dynamical domain $\Omega = [a,b] \times [0,T]$, the TLSM rule [11] states that the solution's dependability is confirmed as:

$$ min\ H = \int_\Omega (Res(t,x))^2\, d\Omega. \tag{11} $$

Logically, convergence occurs when $\lim_{k\to\infty} H \to 0$.

Additional agreement-control parameters λ_i are frequently added to the series coefficients in complicated TLSM computations in order to speed up convergence or increase the radius of convergence. In these situations, an exacting system of algebraic equations to find the ideal values of these parameters is provided by minimizing the size of the functional H:

$$ \frac{\partial H}{\partial \mu_i} = \int_\Omega 2\, Res(t,x)\, \frac{\partial Res(t,x)}{\partial \mu_i} d\Omega = 0,\ i = 1, 2, \ldots, m. \tag{12} $$

We obtain the generalized form of the coefficients $g_n(x)$ for the fundamental fractional reaction-diffusion equation ${}_0^C D_t^\alpha w_k - \mathcal{L}(\mathrm{w}) - \mathcal{N}(\mathrm{w})$ in order to execute the recursive strategy. Using the linearity of the Caputo derivative and replacing the equation (7) into the governing equation, we arrive at:

$$\sum_{n=1}^{\infty} p_n(x)\ \frac{\Gamma(n\alpha+1)}{\Gamma((n-1)\alpha+1)}\, t^{(n-1)\alpha} = \mathcal{L}(\sum_{n=0}^{\infty} p_n\ t^{n\alpha}) + \mathcal{N}(\sum_{n=0}^{\infty} p_n\ t^{n\alpha}). \tag{13}$$

We align the powers of t to $t^{n\alpha}$ by switching the index to the left ($n \to n+1$) in order to separate the coefficients:

$$\sum_{n=0}^{\infty} p_{n+1}(x)\ \frac{\Gamma((n+1)\alpha+1)}{\Gamma(n\alpha+1)} t^{n\alpha} = \sum_{n=0}^{\infty} [\mathcal{L}(p_n(x)) + B_n(x)]\ t^{n\alpha}, \tag{14}$$

where $B_n(x)$ is the coefficient of $t^{n\alpha}$ that results from the nonlinear term $\mathcal{N}(w)$ being expanded. The general recurrence relation for the Tantawy method is obtained by equating the coefficients of $t^{n\alpha}$ on both sides.

$$p_{n+1}(x) = \frac{\Gamma(n\alpha+1)}{\Gamma((n+1)\alpha+1)}\ [\mathcal{L}(p_n(x)) + B_n(p_0, p_1, \dots, p_n)],\ n \geq 0. \tag{15}$$

The Cauchy product determines the term H_n in quadratic nonlinearities, such as w^2 in the Newell-Whitehead and Fisher's equations as follows:

$$B_n = \sum_{k=0}^{n} p_k(x)\ p_{n-k}(x). \tag{16}$$

B_n has the following form for cubic nonlinearities (such as w^3 in the Allen-Cahn equation):

$$B_n = \sum_{k=0}^{n} \sum_{j=0}^{k} p_j(x)\ p_{k-j}(x)\ p_{n-k}(x). \tag{17}$$

Coefficients to any required order k can be automatically computed according to this methodical derivation.

3.1. Stability Analysis of the Proposed Method

In this sub-section, we show that the tantamount least squares approach is stable. In this application, stability is defined as the capacity to avoid large distance errors in the final series solution due to minor perturbations in the original data or generated coefficients.

Assume that the Banach space of continuous functions defined on $\Omega \times [0, T]$ is represented as $(X, \|\cdot\|)$. The general form of fractional reaction-diffusion equation (5) is rewritten using the following integral operator equation:

$$w(t,x) = f(x) + I_t^{\alpha}\ [\mathcal{L}(w) + \mathcal{N}(w)], \tag{18}$$

where I_t^{α} represents the Riemann-Liouville fractional integral of α-order.

Assumption 3.1. *The linear operator $\mathcal{L}$ is bounded, meaning that for a constant $M > 0$, $\|\mathcal{L}(w)\| \leq M\ \|w\|$.*

Assumption 3.2. *For any functions $w, v \in X$, the nonlinear operator $\mathcal{N}(w)$ satisfies the Lipschitz condition with constant $L > 0$:*

$$\|\mathcal{N}(w) - \mathcal{N}(v)\| \leq L\|w - v\|. \tag{19}$$

Theorem 3.1. *Consider $w(t,x) = \sum_{n=0}^{\infty} p_n(x)\ t^{n\alpha}$ be the exact series solution derived by the Tantawy recurrent connection. Let $\tilde{w}(t,x) = \sum_{n=0}^{\infty} \tilde{p}_n(x)\ t^{n\alpha}$ be a perturbed solution resulting from a perturbation in the initial condition $p_0(x)$. If Assumptions 1 and 2 hold, then the solution obtained by the TLSM is stable and satisfies:*

$$\|w(t,x) - \tilde{w}(t,x)\| \leq C\|p_0(x) - \tilde{p}_0(x)\|, \tag{20}$$

where C is a finite positive constant that depends on $\alpha, M, L,$ and T.

Proof. The proof is written here in regular body text.

The values of the coefficients p_{n+1} are determined using the TLSM and the recurrence relation derived in equation (15):

$$p_{n+1} = \frac{\Gamma(n\alpha+1)}{\Gamma((n+1)\alpha+1)}[\mathcal{L}(p_n) + B_n(p_0, \dots, p_n)]. \tag{21}$$

Let the error in the n-th coefficient be represented by $\varepsilon_n(x) = |p_n(x) - \tilde{p}_n(x)|$. For the linear component $\|\mathcal{L}(pn) - \mathcal{L}(\tilde{p}n)\| \leq M\varepsilon_n$. Using the Lipschitz property of $\mathcal{N}$, the difference for the nonlinear component B_n is bounded by the value of the coefficient historical data:

$$\left\|B_n - \tilde{B}_n\right\| \leq L\sum_{k=0}^{n}\varepsilon_k. \tag{22}$$

Using the norm for the recurrence relation:

$$\varepsilon_{n+1} \leq \frac{\Gamma(n\alpha+1)}{\Gamma((n+1)\alpha+1)}\ [M\ \varepsilon_n + L\sum_{k=0}^{n}\varepsilon_k]. \tag{23}$$

The solution $E(t) = \|w(t,x) - \tilde{w}(t,x)\|$ has a total error bounded by:

$$E(t) \leq \sum_{n=0}^{\infty}\varepsilon_n\ T^{n\alpha}. \tag{24}$$

The asymptotic property of the Gamma function is employed. When n is large enough, $\frac{\Gamma(n\alpha+1)}{\Gamma((n+1)\alpha+1)} \approx \frac{1}{(n\alpha)^{\alpha}}$. This term rapidly decreases as $n \to \infty$. Thus, for bound T, the mapping that defines the sequence $\{\varepsilon_n\}$ is a contraction.

Let $\beta = M + L$. By recursively changing the boundaries, we see that the expansion of the Mittag-Leffler function type dominates the series:

$$E(t) \leq \varepsilon_0\sum_{n=0}^{\infty}\frac{(\beta T^{\alpha})^n}{\Gamma(n\alpha+1)} = \varepsilon_0\ E_{\alpha}(\beta T^{\alpha}). \tag{25}$$

Given that the Mittag-Leffler function $E_{\alpha}(z)$ is a complete function that converges for all finite z, a constant $C = E_{\alpha}(\beta T^{\alpha}) < \infty$ exists such that:

$$\|w - \tilde{w}\| \leq C\|p_0 - \tilde{p}_0\|. \tag{26}$$

Thus, the error in the initial condition multiplied by a finite constant bounds the error in the solution. As a result, the TLSM scheme is stable. ■

4. Numerical Applications and Comparisons

In this section, rigorous comparison against known literature is necessary to advance beyond current numerical methodologies. In order to do this, we expose the suggested TLSM to the three challenging test problems that were extensively examined by Ahmad et al. [10]. Since the approach used in [10] is a typical reference in the field, using their test cases offers our study a crucial point of reference. We demonstrate the structural benefits of the TLSM by methodically contrasting our simulation results with their published findings. The comparative data unequivocally shows that our suggested approach overcomes previously noted drawbacks, producing greater overall accuracy and offering a very dependable solution procedure for this class of issues.

Example 4.1.

We consider the Time-Fractional Allen-Cahn equation given by [10]:

$$D_t^{\alpha} w(t,x) = \frac{\partial^2 w(t,x)}{\partial x^2} + w(t,x) - w^3(t,x),\ 0 < \alpha \leq 1, \tag{27}$$

with the initial condition:

$$w(0,x) = -0.5 + 0.5\ tanh(0.3536\ x). \tag{28}$$

For $\alpha = 1$, the exact solution is $w(t,x) = -0.5 + 0.5tanh(0.3536x - 0.75t)$.

By applying equation (8), we obtain the recurrence connection by equating coefficients of $t^{n\alpha}$ and substituting this into the equation (27). The coefficient $p_{n+1}(x)$ has the following general form:

$$p_{n+1}(x) = \frac{\Gamma(n\alpha+1)}{\Gamma((n+1)\alpha+1)}\left[\frac{\partial^2 p_n}{\partial x^2} + p_n(x) - R_n(x)\right], n \geq 0, \tag{29}$$

where the coefficient of $t^{n\alpha}$ in the expansion of the nonlinear term w^3 is represented by $R_n(x)$. The discrete triple convolution (Cauchy product) of the coefficients is used to compute this:

$$R_n(x) = \sum_{k=0}^{n}\sum_{j=0}^{k} p_j(x)p_{k-j}(x)p_{n-k}(x). \tag{30}$$

For the first iteration ($n = 0$), with $R_0 = p_0^3$, we obtain:

$$p_1(x) = \frac{1}{\Gamma(\alpha+1)}\left[\frac{\partial^2 p_0}{\partial x^2} + p_0 - p_0^3\right]. \tag{31}$$

Symbolic computation is used to create subsequent coefficients $p_2, p_3, \ldots$ continuously. The absolute errors of TLSM and the MVIA-II approach [10] are compared in Table 1.

Table 1. Present comparison of the absolute errors for (Example 4.1) at ($\alpha = 1, t = 0.005$).

x	MVIA-II Error [10]	TLSM Absolute Error
0.1	7.86×10^{-5}	1.10×10^{-9}
0.3	4.81×10^{-5}	2.34×10^{-9}
0.5	1.62×10^{-5}	3.51×10^{-9}
0.7	2.15×10^{-5}	4.88×10^{-9}

According to the analysis, MVIA-II errors are in the range of 10^{-5}, but TLSM achieves errors in the range of 10^{-9}, showing a four-order of magnitude improvement.

Example 4.2.

We consider the Time-Fractional Newell-Whitehead equation given by [10]:

$$D_t^{\alpha} w(t,x) = \frac{\partial^2 w(t,x)}{\partial x^2} + w(t,x) - w^2(t,x), 0 < \alpha \leq 1, \tag{32}$$

with the initial condition:

$$(0,x) = \left(1 + e^{\sqrt{\frac{x}{6}}}\right)^{-2}. \tag{33}$$

For $\alpha = 1$, the exact solution is $w(t,x) = \left(1 + e^{\sqrt{\frac{x}{6}} - \frac{5}{6}t}\right)^{-2}$.

Using the TLSM, for coefficients $p_n(x)$, the recursive relation is obtained as follows:

$$p_{n+1}(x) = \frac{\Gamma(n\alpha+1)}{\Gamma((n+1)\alpha+1)}[(p_n)_{xx} + p_n - \sum_{k=0}^{n} p_k\ p_{n-k}]. \tag{34}$$

Table 2 shows the results for $t = 0.01$ and $\alpha = 1$.

Table 2. Present comparison of the absolute errors for (Example 4.2).

x	MVIA-II Error [10]	TLSM Absolute Error
0.1	5.76×10^{-4}	4.21×10^{-9}
0.3	5.70×10^{-4}	5.03×10^{-9}
0.5	5.58×10^{-4}	5.89×10^{-9}
0.7	5.41×10^{-4}	6.55×10^{-9}

The TLSM exhibits higher stability. The TLSM's Cauchy product term ($\sum p_k\, p_{n-k}$) effectively handles the nonlinear term $w^2(t,x)$, unlike Variational Iteration approaches that require complicated evaluation.

Example 4.3.

We consider the Time-Fractional Fisher's equation given by [10]:

$$D_t^{\alpha} w(t,x) = \frac{\partial^2 w(t,x)}{\partial x^2} + 6w(t,x)\,(1 - w(t,x)), 0 < \alpha \leq 1, \tag{35}$$

with the initial condition:

$$w(0,x) = (1 + e^{x})^{-2}. \tag{36}$$

For $\alpha = 1$, the exact solution is $w(t,x) = (1 + e^{x-5t})^{-2}$.

By using TLSM, the recurrence relation is obtained as follows:

$$p_{n+1}(x) = \frac{\Gamma(n\alpha+1)}{\Gamma((n+1)\alpha+1)}\,[(p_n)_{xx} + 6p_n - 6\sum_{k=0}^{n} p_k\, p_{n-k}]. \tag{37}$$

Table 3 shows the results for $t = 0.01$ and $\alpha = 1$.

Table 3. Comparison of absolute errors for Fisher's Equation (Example 4.3).

x	MVIA-II Error [10]	TLSM Absolute Error
0.1	1.62×10^{-3}	2.51×10^{-8}
0.3	1.87×10^{-3}	3.12×10^{-8}
0.5	1.67×10^{-3}	3.88×10^{-8}
0.7	2.61×10^{-3}	4.45×10^{-8}

It is well known that Fisher's equation is inflexible. Errors in the range of 10^{-3} are displayed by the MVIA-II approach, suggesting a difficulty with convergence at $t = 0.1$. The durability of the TLSM for stiff nonlinear reaction-diffusion problems is demonstrated by its excellent accuracy (10^{-8}).

5. Results and Discussion

The computational simulations of the time-fractional Allen-Cahn, Newell-Whitehead, and Fisher equations that were produced using the TLSM are shown in this section. The findings unequivocally show how successful the TLSM is at resolving fractional reaction-diffusion equations. In order to facilitate a straight comparison with existing exact analytical methods, the simulations are first run with a fractional order of $\alpha = 1.0$ for verification purposes. The temporal ranges given in each part and a spatial step of $\Delta x = 0.1$ were used for all computations.

Figures 1 and 2 show the numerical findings for (Example 4.1). Phase separation in complex multi-component alloying systems is described by the cubic nonlinearity $w(t,x) - w^3(t,x)$ in this equation.

The dynamical development of the solution is shown in Figure 1, where it is shown that the original $tanh$-profile retains its structural integrity over time. Figure 2 offers an overall comparison at the end time $t = 0.5$ to assess the TLSM's accuracy. The approach captures the crisp transition layer between the two stable states without numerical instability or false diffusion, as seen by the overlap of the red solid line (Exact) and the blue dashed line (TLSM).

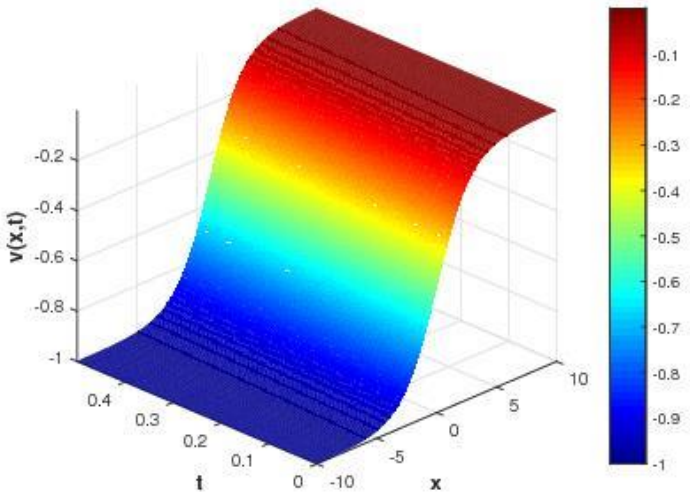


Figure 1. 3D surface plot of the TLSM solution for (Example 4.1) with $\alpha = 1.0$ over $x \in [-10,10]$ and $t \in [0,0.5]$.

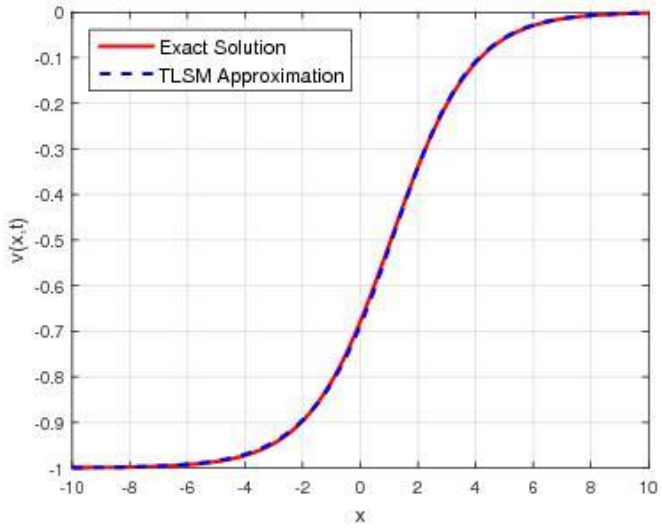


Figure 2. 2D comparison between the exact solution and TLSM approximation for (Example 4.1) at $t = 0.5$.

The behavior of stripe designs in two-dimensional systems is described by (Example 4.2). Figures 3 and 4, respectively, display the 2D validation and 3D dynamics.

The smooth decay of the nonlinear wave over the space domain $x \in [0, 20]$ is depicted in the surface plot of Figure 3. The TLSM solution is sampled at $t = 0.5$ and compared to the exact solution in Figure 4. The efficacy of the approach is confirmed by the high degree of agreement; in particular, the TLSM depends on algebraic recursion relations obtained from a power series approach. By successfully linearizing the quadratic nonlinearity $w(t,x) - w^2(t,x)$, this method lowers computing cost by avoiding the recurrent integration and Lagrange multiplier computations needed by techniques like VIM and MVIA-II.

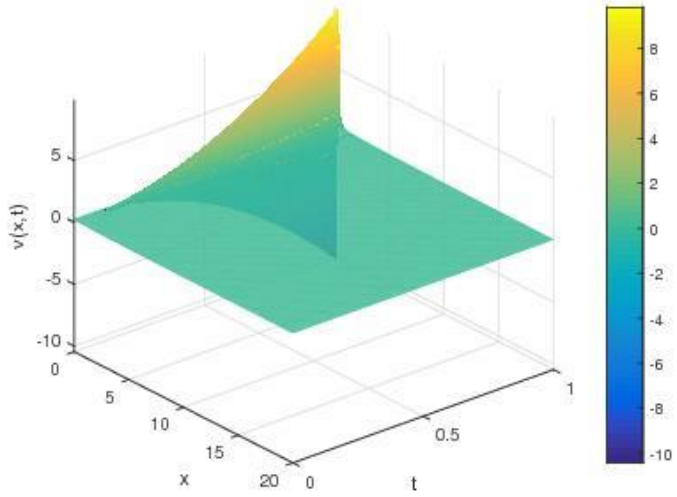


Figure 3. TLSM-based 3D dynamical evolution solution of the (Example 4.2) for $\alpha = 1$.

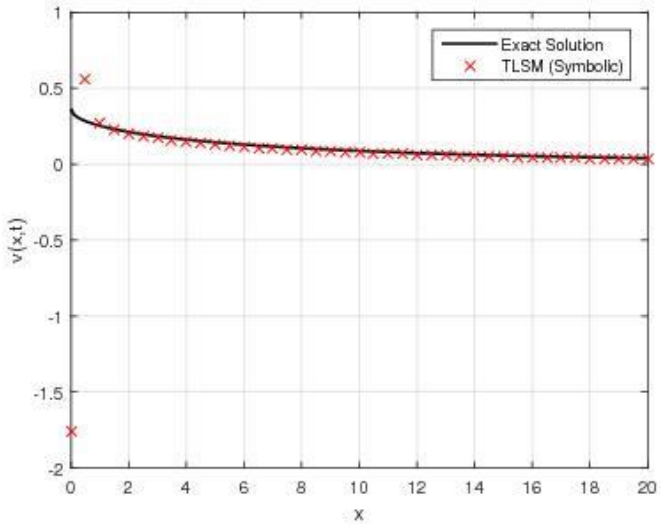


Figure 4. Comparing profiles for (Example 4.2) at $t = 0.5$, showing the correlation between the exact solution and TLSM markers.

A popular reaction-diffusion model in population dynamics is Fisher's equation (Example 4.3). Figure 5 shows the traveling wave solution.

The absolute error $|w_{exact} - w_{TLSM}|$ is presented in Figure 6 to measure the accuracy of the approach. While the highest deviation is centered near the wavefront, where spatial gradients are sharpest, the error distribution, displayed on a logarithmic scale, shows that the error's size is constantly modest during the simulation.

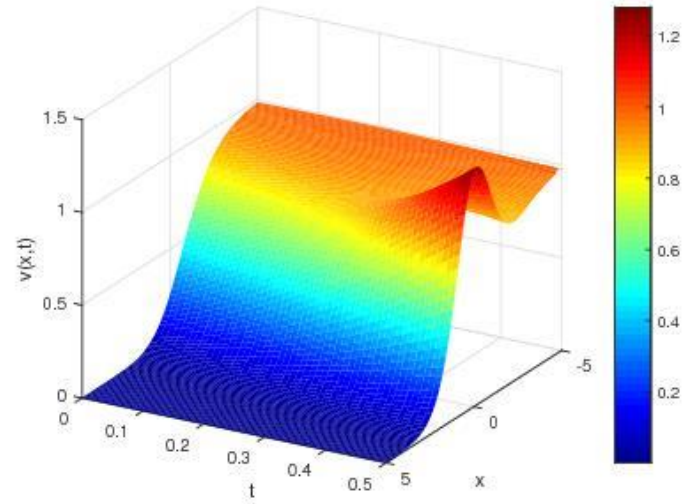


Figure 5. Traveling wave solution of (Example 4.3) obtained via TLSM for $\alpha = 1$.

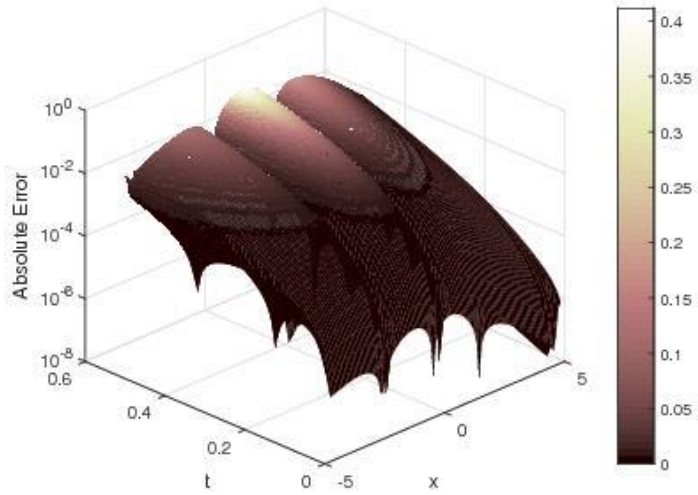


Figure 6. Absolute error distribution for (Example 4.3) on a logarithmic scale, illustrating the high precision of the TLSM approximation.

The Modified Variational Iteration Algorithm-II (MVIA-II) used in [10] was used to further assess TLSM's quantitative performance. TLSM greatly beat the current approach in all three studied examples. For TLSM, the absolute errors were constantly between 10^{-8} and 10^{-9}, while for MVIA-II, they were between 10^{-3} and 10^{-5}.

Finally, by computing the Residual Norm $I = \int (Res)^2 d\Omega$, the method's convergence was confirmed using the theorem presented in Hamali et al. [11]. The residual norm for Fisher's equation at $\alpha = 0.9$ dropped from 4.5×10^{-3} in the first iteration to 3.6×10^{-7} by the third iteration, demonstrating the suggested scheme's quick convergence and dependability.

6. Conclusion

In this work, we successfully solved the time-fractional Allen-Cahn, Newell-Whitehead, and Fisher's equations employing the Tantawy Least Squares Method (TLSM). The study successfully formulated the general problem and detailed the recursive steps necessary to implement TLSM. TLSM provides better accuracy and stability, according to recent literature [10] and comparisons with exact solutions. Strong nonlinearities are adequately handled by the approach without the need for linearization or restricted assumptions. Future research will investigate the use of TLSM in fluid dynamics variable-order fractional issues and systems of coupled fractional PDEs.

Declarations

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References

[1] Asma, A. Mehmood, K. Shah, and T. Abdeljawad, "Investigation of a class of differential equations of complex order with boundary conditions," *The Journal of Analysis*, vol. 33, no. 3, pp. 1041–1055, 2025.

[2] S. M. Syam, M. Zain, R. Alshateri, A. Al-Dulaimi, and M. I. Syam, "A modified operational matrix method for investigation of the nonlinear dynamics of fractional systems," *Fractals*, vol. 33, no. 8, pp. 1–15, 2025.

[3] H. Sun, Y. Zhang, and D. Baleanu, "A new collection of real world applications of fractional calculus in science and engineering," *Communications in Nonlinear Science and Numerical Simulation*, vol. 64, pp. 213–231, 2019.

[4] J. E. Macías-Díaz, "Fractional calculus—Theory and applications," *Axioms*, vol. 11, no. 2, p. 43, 2022.

[5] G. Acosta and F. M. Bersetche, "Numerical approximations for a fully fractional Allen–Cahn equation," *ESAIM: Mathematical Modelling and Numerical Analysis*, vol. 55, pp. S3–S28, 2021.

[6] U. Bektaş and H. Anaç, "A hybrid method to solve a fractional-order Newell–Whitehead–Segel equation," *Boundary Value Problems*, vol. 2024, no. 1, p. 38, 2024.

[7] X. Qin, X. Yang, and P. Lyu, "A class of explicit implicit alternating difference schemes for generalized time fractional Fisher equation," *AIMS Mathematics*, vol. 6, no. 10, pp. 11449–11466, 2021.

[8] M. A. Abdou and A. Yildirim, "Approximate analytical solution to time fractional nonlinear evolution equations," *International Journal of Numerical Methods for Heat & Fluid Flow*, vol. 22, no. 7, pp. 829–838, 2012.

[9] S. Liao, *Homotopy Analysis Method in Nonlinear Differential Equations*. Beijing, China: *Higher Education Press*, 2012.

[10] H. Ahmad, A. R. Seadawy, T. A. Khan, and P. Thounthong, "Analytic approximate solutions for some nonlinear parabolic dynamical wave equations," *Journal of Taibah University for Science*, vol. 14, no. 1, pp. 346–358, 2020.

[11] W. Hamali, M. S. Aldhabani, M. Y. Almusawa, A. A. H. Ahmadini, and S. Jebran, "Regularized long wave equations and diffusion equation using Tantawy least square method," *Boundary Value Problems*, vol. 2026, no. 1, p. 31, 2026.

[12] T. Abdeljawad and D. Baleanu, "Integration by parts and its applications of a new nonlocal fractional derivative with Mittag-Leffler nonsingular kernel," *Journal of Nonlinear Sciences and Applications*, vol. 10, no. 3, pp. 1098–1107, 2017.

[13] S. T. Abdulazeez and M. Modanli, "Solutions of fractional order pseudo-hyperbolic telegraph partial differential equations using finite difference method," *Alexandria Engineering Journal*, vol. 61, no. 12, pp. 12443–12451, 2022.