Sigma Journal of Mathematical and Statistical Sciences (SJMSS)
Published by Altech Press • https://sjmss.altechpress.com
Vol. 1, No. 1 (2026), pp. 21–30

Research Article

Comparative Analysis of Laplace Transform Collocation and Finite Difference Methods for Pseudo-Hyperbolic Telegraph Partial Differential Equations

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Received: 15 August 2026 • Revised: 11 September 2026 • Accepted: 21 September 2026 • Published online: 30 September 2026

ABSTRACT

In this study, an examination is conducted on a pseudo hyperbolic telegraph partial differential equation using two effective methods: the Laplace transform Collocation method and the finite difference technique. The approximations obtained from both methods are compared, and the collected results are presented in an error analysis table. Additionally, a stability estimate is provided for the problem being investigated. The accrued numerical approximate solutions show that the finite difference technique is more appropriate and gets better results for the proposed problem than the Laplace transform Collocation approach since it is closer to the exact solution. Finally, the obtained results for both approaches are graphically represented.

Keywords: Pseudo-hyperbolic telegraph equations; Laplace transform Collocation method; Finite difference method; Numerical solution; Error estimates.

MSC 2020: 35L82; 65M06; 65M70; 65M15.

1. Introduction

Hyperbolic partial differential equations come upon applications in various fields of engineering and science, including fluid dynamics, wave propagation, random motion theory, hydrodynamics, thermal motion, electromagnetics, and materials science. The hyperbolic form of partial differential equations was developed to evaluate engineering and physical science models that often-included physical rules such as Newton's law of balance of forces. Methods for overcoming these challenges have been developed in recent years, there are many methods for solving hyperbolic partial differential equations, as well as numerical and exact solutions [1-6].

Telegraph equations play an essential role in the study of hyperbolic partial differential equations and arise widely in engineering and the applied sciences, where they describe reaction–diffusion phenomena more realistically than classical diffusion equations owing to their finite propagation speed. Their applications range from random motion theory, where fractional and heterogeneous telegraph processes with memory and stochastic resetting have recently been analyzed [7, 10], to electromagnetic energy transfer studied through multiphysics simulation of microwave heating [8], and to signal propagation along transmission lines, for which Cvetićanin et al. derived non-local telegrapher's equations as a generalized transmission line model [9]. Among the many numerical strategies proposed in recent years, Hashemi et al. developed high-order compact finite difference methods for the multidimensional hyperbolic telegraph equation with rigorous stability and convergence analysis [11].

The pseudo hyperbolic telegraph partial differential equation is a complex mathematical system that involves mixed partial derivatives. This equation, which is widely used in various fields including thermoelectricity, plasma physics, nerve conduction, longitudinal vibrations, and response diffusion, has been extensively researched in several publications. These studies have focused on different aspects of the equation, such as the existence and uniqueness of numerical solutions [12], stability estimates of numerical solutions [13], and the existence and uniqueness of solutions for Cauchy-type problems [14].

On the other hand, the Laplace transform Collocation method and the finite difference approach are two robust and precise numerical techniques for calculating the approximation solution of the pseudo hyperbolic telegraph equations. The Laplace transform Collocation method is an effective technique based on the Laplace transform. Additionally, the finite difference method and its approximation solution, stability estimate, and theoretical benefits are crucial in the field of mathematics. Therefore, it is crucial to understand the distinctions between them concerning precision, reliability, and computational duration. The Laplace transform Collocation technique and the finite difference method are two robust and efficient approaches utilized to determine both approximate and analytical solutions for a wide range of problem types, for example, the authors in [15] presented the Laplace Collocation technique for solving some types of differential equation in the Caputo sense. With the help of the Laplace Transform Collocation Method, the authors in [16] were able to find a rough solution to one-dimensional telegraph equations. By utilizing the Daftar-Gejii-Jafaris approach and the Laplace transform Collocation method, the author in [17] was able to approximate the solution of fractional order time-varying dynamical system in sense of the Caputo derivative. Finally, in [18], researchers employed the explicit finite difference method to acquire an approximate solution to the pseudo-hyperbolic equations, utilizing the Caputo fractional derivative.

Building upon the numerical framework established by Modanli and Özbağ (2022) for the pseudo-hyperbolic telegraph equation [19], this study extends the application of the finite difference scheme to a distinct problem. Furthermore, to thoroughly evaluate the performance and reliability of the proposed approach, the numerical findings are compared with those obtained via the Laplace Transform Collocation method.

The following partial differential equations for the boundary value problem will be examined in this study:

$$\begin{cases} \frac{\partial^2 w(t,x)}{\partial t^2} + \frac{\partial w(t,x)}{\partial t} + w(t,x) = \beta \frac{\partial^3 w(t,x)}{\partial x^2 \partial t} + \frac{\partial^2 w(t,x)}{\partial x^2} + f(t,x) \\ 0 \le x \le L, \qquad 0 \le t \le T \\ w(0,x) = \varphi(x), \;\; w_t(0,x) = \psi(x), \end{cases} \tag{1}$$

where f, φ and ψ in the defined regions are known continuous functions, β is a constant and $\beta > 0$, and w is an unknown function [20]. The purpose of this research is to find numerical solutions for pseudo-hyperbolic telegraph partial differential equations using two efficient techniques: The Laplace transform Collocation technique and the finite difference approach. By employing the Laplace transform procedure,

a novel explicit function is derived from the fundamental equation. The Laplace transform Collocation method is then utilized to approximate the solutions for the given problem.

2. Laplace Transform Collocation Method (LTCM)

In this section, we present a brief overview of how to apply the Laplace transform Collocation method to solve the pseudo hyperbolic telegraph equations, similar to the approach used in [20].

To begin, we use the Laplace transform on both sides of equation (1), resulting in the following expression:

$$s^2\mathrm{w}(\mathrm{s},\mathrm{x}) - \mathrm{sw}(0,\mathrm{x}) - w_t(0,\mathrm{x}) = -\mathcal{L}\left\{\frac{\partial w(t,x)}{\partial t}\right\} - \mathcal{L}\{w(t,x)\} + \beta\mathcal{L}\left\{\frac{\partial^3 w(t,x)}{\partial x^2 \partial t}\right\} + \mathcal{L}\left\{\frac{\partial^2 w(t,x)}{\partial x^2}\right\} + \mathcal{L}\{f(t.x)\}. \quad (2)$$

The explicit form of the function $w(t,x)$ is applied for Laplace transform Collocation method, then we have the following formula

$$w = w_0 + \sum_{n=1}^{k} c_n\, w_{n,} \quad (3)$$

here c_n, are constant coefficients that can be obtained using the initial values from equation (1). When we apply equation (3) to equation (2), we get

$$W(s,x) = \frac{1}{s^2}\left\{\mathrm{s}\left(w_0(0,x) + \sum_{n=1}^{k} c_n\, w_n(0,x)\right) + \frac{\partial}{\partial t}\left(w_0(0,x) + \sum_{n=1}^{k} c_n\, w_n(0,x)\right) + \mathcal{L}\left\{\frac{\partial}{\partial x^2}\left(w_0(t,x) + \sum_{n=1}^{k} c_n\, w_n(t,x)\right)\right\} + \beta\mathcal{L}\left\{\frac{\partial^3 w(t,x)}{\partial x^2 \partial t}\left(w_0(t,x) + \sum_{n=1}^{k} c_n\, w_n(t,x)\right)\right\} - \mathcal{L}\left\{\frac{\partial w(t,x)}{\partial t}(w_0(t,x) + \sum_{n=1}^{k} c_n\, w_n(t,x))\right\} - (w_0(t,x) + \sum_{n=1}^{k} c_n\, w_n(t,x)) + \mathcal{L}\{f(t,x)\}.$$

By applying the inverse Laplace transform to the previous equation, we get

$$w_{new}(t,x) = \mathcal{L}^{-1}\left[\frac{1}{s^2}\left\{\mathrm{s}\left(w_0(0,x) + \sum_{n=1}^{k} c_n\, w_n(0,x)\right) + \frac{\partial}{\partial t}\left(w_0(0,x) + \sum_{n=1}^{k} c_n\, w_n(0,x)\right) + \mathcal{L}\left\{\frac{\partial}{\partial x^2}\left(w_0(t,x) + \sum_{n=1}^{k} c_n\, w_n(t,x)\right)\right\} + \beta\mathcal{L}\left\{\frac{\partial^3 w(t,x)}{\partial x^2 \partial t}\left(w_0(t,x) + \sum_{n=1}^{k} c_n\, w_n(t,x)\right)\right\} - \mathcal{L}\left\{\frac{\partial w(t,x)}{\partial t}(w_0(t,x) + \sum_{n=1}^{k} c_n\, w_n(t,x))\right\} - \left(w_0(t,x) + \sum_{n=1}^{k} c_n\, w_n(t,x)\right) + \mathcal{L}\{f(t,x)\}\right]. \quad (4)$$

If equation (4) is used in equation (1) at points $x = x_j$, then the new Collocation equation becomes

$$\frac{\partial^2 w_{new}(t,x_j)}{\partial t^2} + \frac{\partial w_{new}(t,x_j)}{\partial t} - \frac{\partial^2 w_{new}(t,x_j)}{\partial x^2} + u_{new}(t,x_j) - \frac{\partial^3 w_{new}(t,x_j)}{\partial x^2 \partial t} = f(t,x_j), \quad (5)$$

here $j = 1,2,\dots,n$ for $x_j = \frac{L-0}{n+1}j$.

The remaining term for this new equation is

$$R_n(t,x) = L[w_n(t,x)] - f(t,x), \quad (6)$$

in the given context, the functions $w(t,x)$ and $w_n(t,x)$ represent the precise and estimated solutions correspondingly, and are expressed in the following manner

$$L[w_n(t,x)] = \frac{\partial^2 w_n(t,x)}{\partial t^2} + \frac{\partial w_n(t,x)}{\partial t} - \frac{\partial^2 w_n(t,x)}{\partial x^2} - \frac{\partial^3 w_n(t,x)}{\partial x^2 \partial t} + w_n(t,x), \quad (7)$$

we can write (7), as

$$\frac{\partial^2 w_n(t,x)}{\partial t^2}+\frac{\partial w_n(t,x)}{\partial t}-\frac{\partial^2 w_n(t,x)}{\partial x^2}+w_n(t,x)=f+R_n, \tag{8}$$

taking L as a linear operator and $e_n(t,x)=w(t,x)-w_n(t,x)$, we get

$$\frac{\partial^2 e_n(t,x)}{\partial t^2}+\frac{\partial w_n(t,x)}{\partial t}-\frac{\partial^2 e_n(t,x)}{\partial x^2}-\frac{\partial^3 w_n(t,x)}{\partial x^2\partial t}+e_n(t,x)=R_n(t,x). \tag{9}$$

By using the initial value conditions in equation (1), then we obtain the new initial value conditions for equation (9) in the form

$$e_n(0,x)=(e_{n)_t}(0,x)=0, \tag{10}$$

$$e_n(t,0)=e_n(t,L)=0, \tag{11}$$

by solving equation (9) and using the initial values (10) and (11), the equation $e_n(t,x)$ is obtained as: $e_n(t,x)=w_n(t,x)-w(t,x)$.

3. Applications of the Laplace Transform Collocation Method

In this section, we will present examples of how the Laplace transform Collocation method will be applied to solve the pseudo hyperbolic telegraph equation. For this purpose, the following pseudo hyperbolic telegraph partial differential equations is considered

$$\begin{cases}\frac{\partial^2 w(t,x)}{\partial t^2}+\frac{\partial w(t,x)}{\partial t}+w(t,x)=\beta\frac{\partial^3 w(t,x)}{\partial x^2\partial t}+\frac{\partial^2 w(t,x)}{\partial x^2}+\frac{4}{3}e^{-t}sinx,\\ w(0,x)=sinx,\ \ w_t(0,x)=-sinx,\\ 0\le x\le\pi,\ \ 0\le t\le 1.\end{cases} \tag{12}$$

To find an approximate solution for this problem, we will apply the Laplace transform Collocation method. By taking the Laplace transform of both sides of equation (12), we can derive the following result.

$$w(s,x)=\frac{1}{s^2}\Big[s.sinx-sinx-\mathcal{L}\Big\{\frac{\partial w(t,x)}{\partial t}\Big\}-\mathcal{L}\{w(t,x)\}+\frac{2}{3}\mathcal{L}\Big\{\frac{\partial^3 w(t,x)}{\partial x^2\partial t}\Big\}+\mathcal{L}\Big\{\frac{\partial^2 w(t,x)}{\partial x^2}\Big\}+\frac{4}{3}\mathcal{L}\{e^{-t}sinx\}\Big], \tag{13}$$

by employing the Laplace transform Collocation method to equation (12), we derive the subsequent expression representing an estimation for the solution

$$w(t,x)=sinx-tsinx+C_1t^2(x^2(x-\pi))+C_2t^2x(x-\pi)^2. \tag{14}$$

By using formula (8), we get

$$\begin{aligned}R(t,x,C_1,C_2)=&\Big[\Big(-2t-2t^2-\frac{1}{3}t^3\Big)(C_1+C_2)\Big]\,x^3+\Big[\Big(2t+2t^2+\frac{1}{3}t^3\Big)(C_1+2C_2)\Big]\pi x^2+\\ &\Big[-\pi\Big(2t+2t^2+\frac{1}{3}t^3\Big)C_2+\Big(8t+16t^2+\frac{16}{3}t^3+\frac{1}{2}t^4\Big)(C_1+C_2)\Big]\,x+\Big(-\frac{4}{3}t+\Big(-1-\frac{2}{3}-\frac{2}{3}\Big)t^2+\\ &\Big(-\frac{1}{3}-\frac{2}{9}-\frac{1}{3}\Big)t^3-\frac{1}{12}t^4\Big)2\pi(C_1+2C_2)+\Big(-\frac{8}{9}e^{-t}-2+t+\Big(1+\frac{2}{3}\Big)t^2+\frac{1}{3}t^3\Big)sinx.\end{aligned} \tag{15}$$

From the equation (15), we obtain

$$(-Ax^3+A\pi x^2+Bx+2C\pi)C_1+(-Ax^3+2A\pi x^2-A\pi x+Bx+4Cx)C_2+Dsinx=0, \tag{16}$$

here, $A=2t+2t^2+\frac{1}{3}t^3$,

$B=8t+14t^2+\frac{16}{3}t^3+\frac{1}{2}t^4$,

$C=-\frac{4}{3}t-\frac{7}{3}t^2-\frac{8}{9}t^3-\frac{1}{12}t^4$,

$$D = -\frac{8}{3}e^{-t} - 2 + t + \frac{5}{3}t^2 + \frac{1}{3}t^3.$$

If we take $x_1 = \frac{1}{2}$, $x_2 = 3$ and $t = \frac{99}{100}$, in the formula (16), we get

$$w(t,x) = sinx - tsinx + C_1t^2(x^2(x-\pi)) + C_2t^2x(x-\pi)^2. \tag{17}$$

4. Finite Difference Method and Stability Analysis

In this section, we will create a finite difference approach and showcase its stability concerning the pseudo-hyperbolic telegraph partial differential equation (1). Initially, we suppose that the grid spacing for the finite difference scheme within the rectangular region $w^h \times w^\tau$ is characterized by $h = \frac{r}{M}$ for the x −axis and $\tau = \frac{v}{N}$ for the t −axis, then we have

$x_n = nh, n = 1,2,\cdots,M, \quad t_k = k\tau, k = 1,2,\cdots,N.$

At the points $(t_k, x_n) \in w^h \times w^\tau$, we can write the system (1) as

$$w_{tt}(t_k,x_n) + w_t(t_k,x_n) + w(t_k,x_n) = \beta w_{txx}(t_k,x_n) + w_{xx}(t_k,x_n) + f(t_k,x_n). \tag{18}$$

The first order difference scheme technique for equation (18) is developed in this stage using the Taylor expansion series with respect to x, which is provided by the following formulas

$$w_{tt}(t_k,x_n) = \frac{w_n^{k+1} - 2w_n^k + w_n^{k-1}}{\tau^2}, \tag{19}$$

$$w_t(t_k,x_n) = \frac{w_n^k - w_n^{k-1}}{\tau}, \tag{20}$$

$$w_{txx}(t_k,x_n) = \frac{1}{\tau}\left(\frac{w_{n+1}^k - 2w_n^k + w_{n-1}^k}{h^2} - \frac{w_{n+1}^{k-1} - 2w_n^{k-1} + w_{n-1}^{k-1}}{h^2}\right), \tag{21}$$

$$w_{xx}(t_k,x_n) = \frac{w_{n+1}^k - 2w_n^k + w_{n-1}^k}{h^2}, \tag{22}$$

the equations (19), (20), (21), and (22) are applied to formulate the difference schemes for equation (18) in the following manner

$$\begin{cases} \frac{w_n^{k+1} - 2w_n^k + w_n^{k-1}}{\tau^2} + \frac{w_n^k - w_n^{k-1}}{\tau} + u_n^k = \frac{\beta}{\tau}\left(\frac{w_{n+1}^k - 2w_n^k + w_{n-1}^k}{h^2} - \frac{w_{n+1}^{k-1} - 2w_n^{k-1} + w_{n-1}^{k-1}}{h^2}\right) \\ \quad + \frac{w_{n+1}^k - 2w_n^k + w_{n-1}^k}{h^2} + f_n^k, \ 1 \le k \le N-1, \ 1 \le n \le M-1, \\ w_n^0 = sinx, \ \frac{w_n^1 - w_n^0}{\tau} = -sinx, \quad 0 \le n \le M, \\ w_0^k = \ w_M^k = 0, \quad 0 \le k \le N. \end{cases} \tag{23}$$

The difference scheme formula (23) can be rewrite by using the initial condition, as follow

$$\begin{cases} \left(\frac{\beta}{h^2\tau}\right)w_{n+1}^{k-1} + \left(-\frac{\beta}{h^2\tau} - \frac{1}{h^2}\right)w_{n+1}^k + \left(\frac{\beta}{h^2\tau}\right)w_{n-1}^{k-1} + \left(-\frac{\beta}{h^2\tau} - \frac{1}{h^2}\right)w_{n-1}^k + \\ \left(\frac{1}{\tau^2} - \frac{1}{\tau} - \frac{2\beta}{h^2\tau}\right)w_n^{k-1} + \left(\frac{-2}{\tau^2} + \frac{1}{\tau} + 1 + \frac{2\beta}{h^2\tau} + \frac{2}{h^2}\right)w_n^k + \left(\frac{1}{\tau^2}\right)w_n^{k+1} = f_n^k \end{cases} \tag{24}$$

Theorem 4.1. *If* $\frac{h^2}{2} + \frac{h^2\tau}{2} + \tau - \frac{h^2}{\tau} > \beta$. *Then the formula (24) is stable.*

Proof. To prove this theorem, if we take the $\sigma = 1$, in formula (13) as in [21], then we get the same system as in formula (24), and the theorem is proven.

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5. Results and Discussion

In this section, we present the numerical outcomes of solving the pseudo-hyperbolic telegraph partial differential equations using the Laplace transform Collocation and finite difference techniques. To obtain the exact solution (as given by equation 12), we employ the modified double Laplace method described in [20]. The exact solution is depicted as $w(t,x) = e^{-t}sinx$. The Laplace transform colocation method was used to find the approximate solution to the proposed model. An approximate solution is obtained by deriving a new explicit function from the fundamental equation utilising the Laplace transform method, as

$$w(t,x) = sinx - tsinx + C_1t^2(x^2(x-\pi)) + C_2t^2x(x-\pi)^2.$$

The error analysis table demonstrates a comparison between the exact solution and the approximate solution of problem (12). The table was constructed using two methods: the Laplace transform Collocation method and the finite difference method. The Matlab software is used to create the data analysis table. We obtain the error estimates because we cannot determine the expected error from the graphical representations.

Table 1. Numerical Outcomes

N, M	Error (LTCM)	Error (FDM)
$\mathrm{N} = 25, \mathrm{M} = 5$	0.389697200443069	0.003224717155240
$\mathrm{N} = 100, \mathrm{M} = 10$	0.002114054886333	7.9835×10^{-4}
$\mathrm{N} = 400, \mathrm{M} = 20$	2.8770×10^{-4}	1.9660×10^{-4}
$\mathrm{N} = 900, \mathrm{M} = 30$	4.6925×10^{-5}	8.7160×10^{-5}
$\mathrm{N} = 1600, \mathrm{M} = 40$	1.2167×10^{-5}	4.8861×10^{-5}

Table 1. Explained the error estimates of equation (12) by using the Laplace transform Collocation technique and the finite difference approach. It is clear that the error analysis columns obtained by both methods are in good agreement.

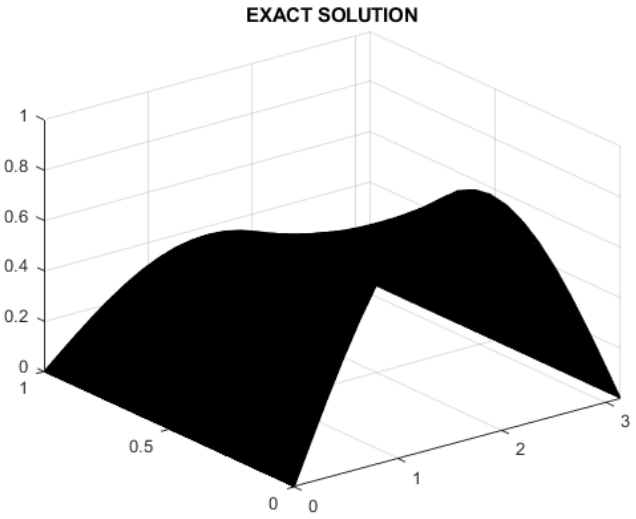


Figure1. Gives the exact solution of (12) for $0 \le x \le \pi,\ \ 0 \le t \le 1$.

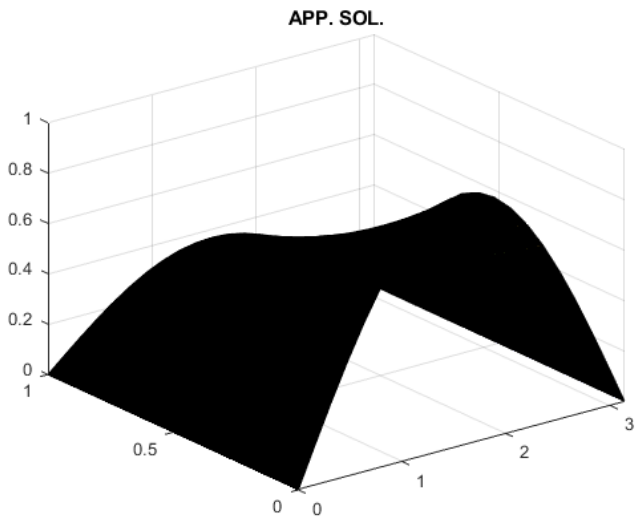


Figure 2. Explain the approximate solution of (12), at $M = 20, N = 400, 0 \leq x \leq \pi$, and $0 \leq t \leq 1$ by using finite difference method.

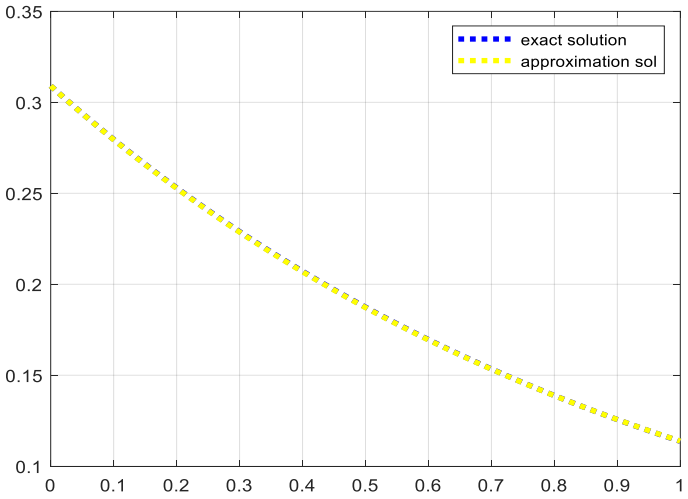


Figure 3. Compares the results of the exact and approximate solutions of (12), $M = 20, N = 400$, and $0 \leq t \leq 1$, by using finite difference method.

The graphical representation of the solution obtained through the finite difference method reveals a remarkable similarity between the exact and approximate solutions, as depicted in Figure 1 and Figure 2. Figure 3 further confirms that there exists a negligible disparity between these two solutions when using the same method.

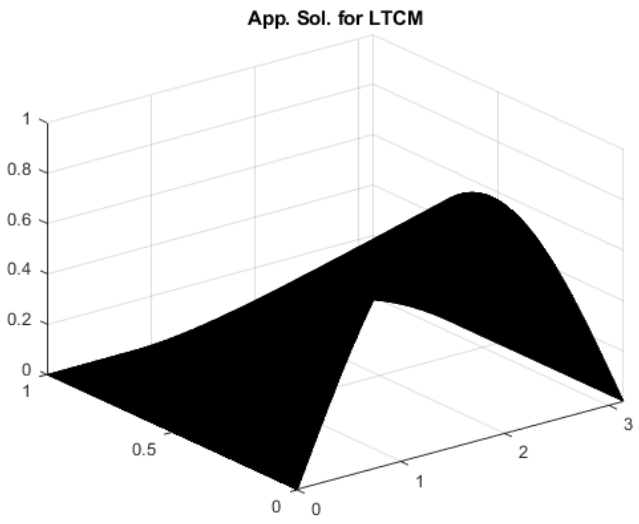


Figure 4. Shows the approximate solution of (12), $M = 20, N = 400, 0 \leq x \leq \pi$, and $0 \leq t \leq 1$, by using Laplace transform colocation method.

The disparity between the exact and approximate solutions is evident in Figures (1) and (4). However, in Figure (5), we observe a close resemblance between the exact and approximate solutions for the regions where $0 \leq x < 0.2$ and $t > 0.06$. Overall, the figures and data analysis table indicate that the finite difference approach outperforms the Laplace transform Collocation strategy when solving the pseudo-hyperbolic telegraph partial differential equation using the proposed methods.

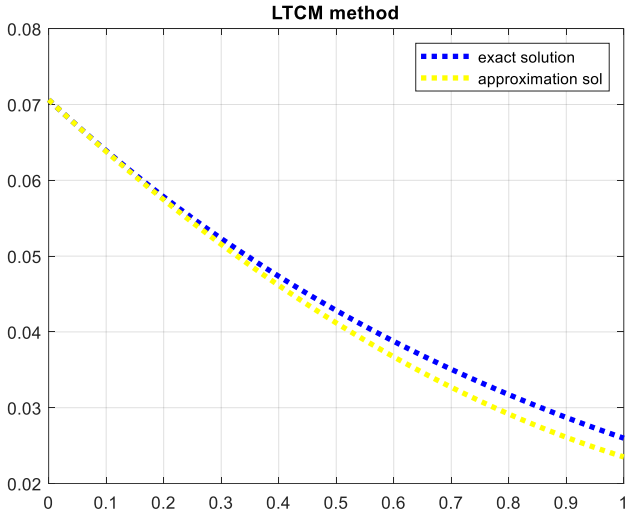


Figure 5. Compare the results of the exact and approximate solutions of equation (12) via the Laplace transform colocation technique.

6. Conclusion

In this study, we investigated the solution of the pseudo-hyperbolic telegraph partial differential equation through the application of two numerical methods: The Laplace transform Collocation method and the finite difference method. Additionally, we obtained the exact solution using a modified Double Laplace transform method. The main objective was to compare and evaluate the accuracy and efficiency of these methods in solving the proposed problem. The results demonstrated that the modified Double Laplace transform method successfully yielded the exact solution for the problem. However, due to its complexity, this method might not be the most practical choice for solving similar problems in real-world applications.

For the approximation solutions, we employed both the Laplace transform Collocation approach and the finite difference method. The findings, including the error analysis presented in Table (1), provided valuable insights into the performance of each method. The comparison revealed that the finite difference method outperforms the Laplace transform Collocation approach in terms of accuracy and robustness for the given problem. FDM deals with local grid points and handles derivative approximations systematically via Taylor expansions, whereas global trial functions in LTCM may experience convergence limitations under certain boundary constraints for this specific pseudo-hyperbolic structure. Furthermore, we performed a stability analysis for the finite difference method, which showed that it remains stable for the given problem domain and initial/boundary conditions. This crucial analysis ensures the reliability of the finite difference method in various practical procedures. One-dimensional issues were the main subject of the investigation. Future research will face additional obstacles and opportunities as the development of extending numerical procedures to solve pseudo-hyperbolic telegraph equations in higher dimensions.

Declarations

Acknowledgments: The authors gratefully acknowledge the support of Universität Duisburg-Essen.

Funding: This work was supported by Universität Duisburg-Essen.

Conflicts of interest: Mahmut Modanlı is a member of the Editorial Board of SJMSS. He was not involved in the editorial handling or peer review of this manuscript..

Author contributions: Mahmut Modanli: methodology, Supervision, writing-review and editing. **Fatma Şimşek:** writing-original draft preparation, methodology, software. **Hazal Yüksekkaya:** Conceptualization, methodology, writing-original draft preparation. **Aysel Ramazanova:** software, writing-review and editing. All authors have read and agreed to the published version of the manuscript.

Data availability: The work did not utilize any data.

Use of AI tools: During the preparation of this work, the authors did not use any generative AI or AI-assisted technologies. The authors take full responsibility for the content of the publication.

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